19/1/22

B.Sc. Part I (Hons.)

2nd Paper

Differential calculus

Partial differentiation

Let $u = x^2 y^3$

Here u is a function of x and y.

$\Rightarrow$ u can be either differentiated (with respect to x) partially or with respect to y (keeping x constant)

Here $u = x^2 y^3$ ——— (1)

Differentiating partially with respect to x, keeping y constant, we get

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}(x^2 y^3) = y^3 \frac{\partial}{\partial x}(x^2)$$

[Note that here y^3 is considered constant.]

$$\Rightarrow \frac{\partial u}{\partial x} = y^3 \cdot 2x = 2xy^3 \cdot \text{——— (2)}$$

$\partial \rightarrow$ Partial differentiation

$d \rightarrow$ total differentiation

$\delta \rightarrow$ increment

$\partial \neq \delta$

Differentiating (2) partially with respect to y, keeping x constant, we get-

$$\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}\right) = \frac{\partial}{\partial y}\left(2xy^3\right) = 2x\frac{\partial}{\partial y}\left(y^3\right)$$

[Here $2x$ is constant

$$= 2x \cdot 3y^2$$

$$\Rightarrow \frac{\partial^2 u}{\partial y \partial x} = 6xy^2. \quad \text{——(3)}$$

Again, $u = x^2y^3$

Differentiating it partially with respect to y, keeping x constant, we get-

$$\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left(x^2y^3\right) = x^2\frac{\partial}{\partial y}\left(y^3\right) = x^2 \cdot 3y^2$$

Now differentiating it partially with respect to x, keeping y constant, we get

$$\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y}\right) = \frac{\partial}{\partial x}\left(x^2 \cdot 3y^2\right)$$

$$= 3y^2\frac{\partial}{\partial x}\left(x^2\right) = 3y^2 \cdot 2x$$

$$= 6xy^2.$$

$$\therefore \frac{\partial^2 u}{\partial x \partial y} = 6xy^2 \quad \text{——(4)}$$

from (3) and (4), $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$.